1.5 BASIC DISCRETE-TIME SIGNALS

Here we consider some simple, standard discrete-time signals.

1.5.1 Unit Step Sequence

Like the continuous-time unit step, we define the unit step sequence $u[n]$ as

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases} \tag{1.42}$$

As shown in Figure 1.24, $u[n]$ is a sequence of 1s starting at the origin. Notice that $u[n]$ is defined at $n = 0$ unlike $u(t)$, which is not defined at $t = 0$.

1.5.2 Unit Impulse Sequence

In discrete time, we define unit impulse sequence as

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \tag{1.43}$$

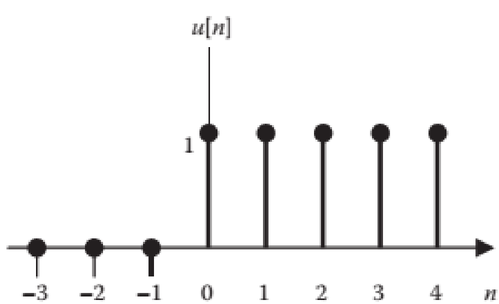


FIGURE 1.24 The unit step sequence.

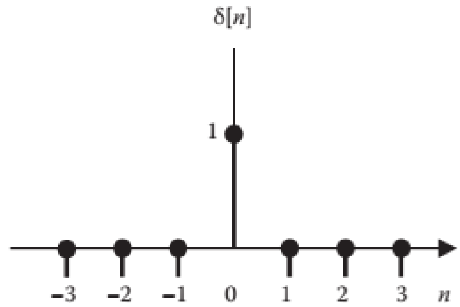


FIGURE 1.25 The unit impulse sequence.

TABLE 1.2
Properties of the Unit Impulse Sequence

1. $x[n]\delta[n] = x[0]\delta[n]$
2. $x[n]\delta[n-k] = x[k]\delta[n-k]$
3. $\delta[n] = u[n]-u[n-1]$
4. $u[n] = \sum_{k=-\infty}^{n} \delta[k]$
5. $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

This is illustrated in Figure 1.25. Notice that we do not have difficulties in defining $\delta[n]$ unlike $\delta(t)$. Some properties of the unit impulse sequence are listed in Table 1.2.

1.5.3 Unit Ramp Sequence

The **unit ramp sequence** is defined as

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases} \tag{1.44}$$

The sequence is shown in Figure 1.26. The relationships between unit impulse, unit step, and unit ramp sequences are

$$\delta[n] = u[n] - u[n-1] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{1.45}$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \tag{1.46}$$

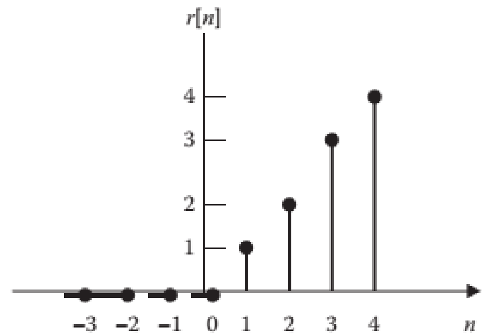


FIGURE 1.26 The unit ramp sequence.

$$u[n] = r[n+1] - r[n] \tag{1.47}$$

$$r[n] = \sum_{m=-\infty}^{n-1} u[m] \tag{1.48}$$

1.5.4 Sinusoidal Sequence

The sinusoidal sequence or a discrete-time sinusoid is given by

$$x[n] = A\cos\left(\frac{2\pi n}{N} + \theta\right) = \mathrm{Re}\left[Ae^{j(2\pi n/N+\theta)}\right] \tag{1.49}$$

where

A is a positive real number and is the amplitude of the sequence
N is the period
θ is the phase
n is an integer

A typical sinusoidal sequence for $A = 1$, $N = 12$, and $\theta = 0$ is shown in Figure 1.27.

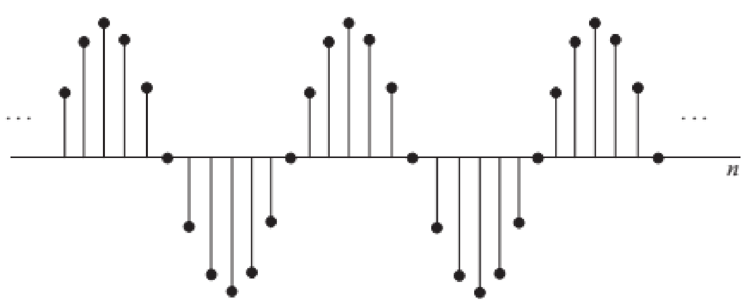


FIGURE 1.27 A discrete-time sinusoidal sequence, $x[n] = \cos(\pi n/6)$.

1.5.5 Exponential Sequence

If we sample a continuous-time exponential function $x(t) = Ae^{-at}$ with sampling period T, we obtain the sequence $x[n] = Ae^{-anT} = A\alpha^n$, with $\alpha = e^{-aT}$. Thus, the exponential sequence is given by

$$x[n] = A\alpha^n \tag{1.50}$$

where
  A and α are generally complex numbers
  n is an integer

A typical discrete-time exponential sequence is shown in Figure 1.28. For the signal shown in Figure 1.28, both A and α are real numbers.

Example 1.7

If $r[n] = nu[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$, Find $y[n] = 2r[1-n]$

Solution
To obtain $y[n]$, we replace every n in $r[n]$ with $-n + 1$.

$$y[n] = 2r[-n+1] = 2(-n+1)u[-n+1] = \begin{cases} -2n+2, & -n+1 \geq 0 \\ 0, & -n+1 < 0 \end{cases}$$

$$= \begin{cases} -2n+2, & n \leq 1 \\ 0, & n > 1 \end{cases}$$

Practice Problem 1.7 Given $r[n]$ in Example 1.7, obtain $z[n] = r(n + 2)$.

Answer: $z[n] = \begin{cases} n+2, & n \geq -2 \\ 0, & n < -2 \end{cases}$

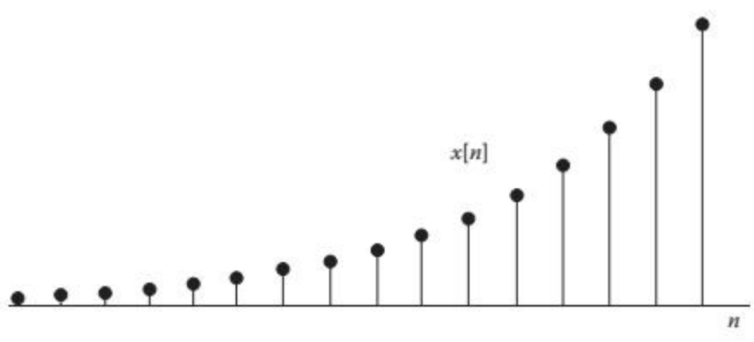
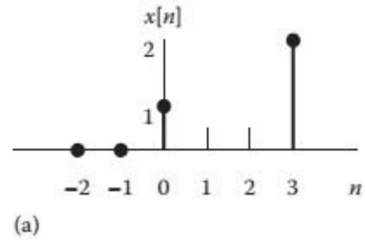


FIGURE 1.28 A discrete-time exponential sequence, $\alpha > 1$.



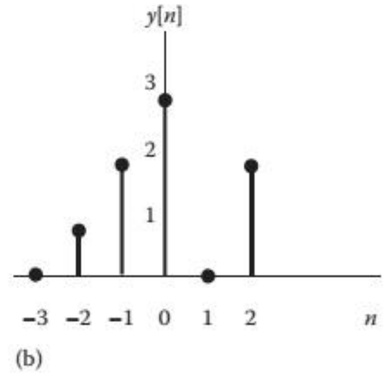


FIGURE 1.29 For Example 1.8.

Example 1.8

Write down expressions for the sequences shown in Figure 1.29.

Solution

We use item 5 in Table 1.2 as a general way of expressing any discrete signal.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

(a) For the discrete-time signal in Figure 1.29a,

$$x[n] = \delta(n) + 2\delta(n-3)$$

(b) Similarly, for $y[n]$ in Figure 1.29b,

$$y[n] = \delta[n+2] + 2\delta[n+1] + 3\delta[n] + 2\delta[n-2]$$

Practice Problem 1.8 Write down expressions for the sequences shown in Figure 1.30.

Answer:

(a) $x[n] = \delta[n + 1] + 2\delta[n-1]$
(b) $y[n] = 2\delta[n + 2] + \delta[n + 1] + \delta[n-2] + 2\delta[n-2]$

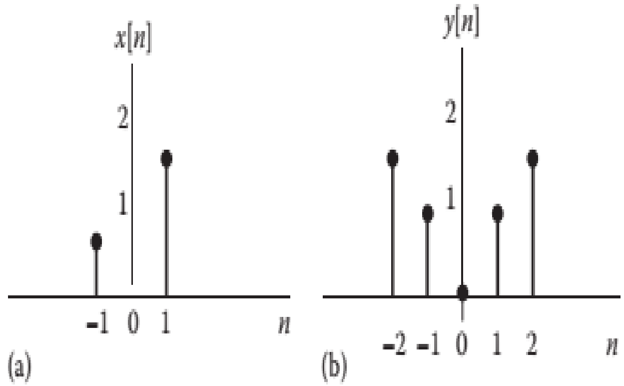


FIGURE 1.30 For Practice Problem 1.8.

Practice Problem 1.10 With the discrete-time signal shown in Figure 1.37, sketch each of the following signals: (a) $x[-n]$, (b) $x[n+2]$, (c) $x[n/2]$

Answer: See Figure 1.39.

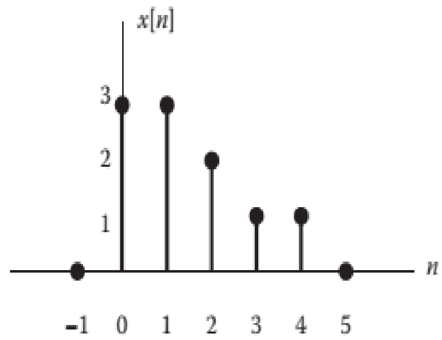


FIGURE 1.37 For Example 1.10.

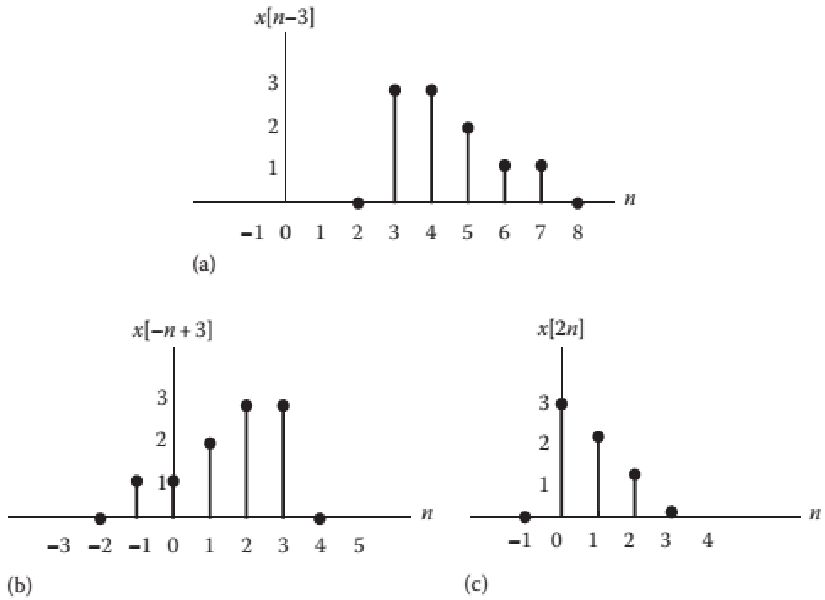


FIGURE 1.38 For Example 1.10.

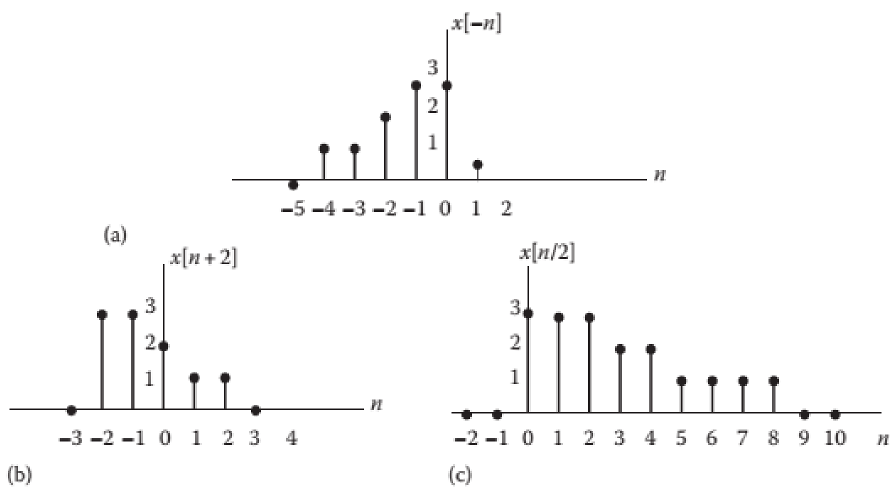


FIGURE 1.39 For Practice Problem 1.10.

1.31 Given the discrete-time signal in Figure 1.61, sketch the following signals:

(a) $y[n] = x[n-3]$

(b) $z[n] = x[n] - x[n-1]$

1.32 Consider the discrete-time signal in Figure 1.62. Sketch the following signals:

(a) $x[n]u[2-n]$

(b) $x[n]\left[u[n+1]-u[n]\right]$

(c) $x[n]\delta[n-2]$

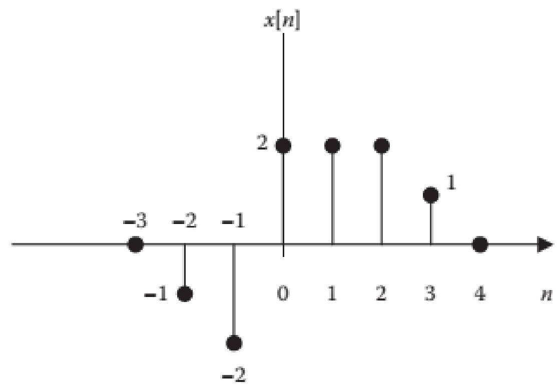


FIGURE 1.61 For Problem 1.31.

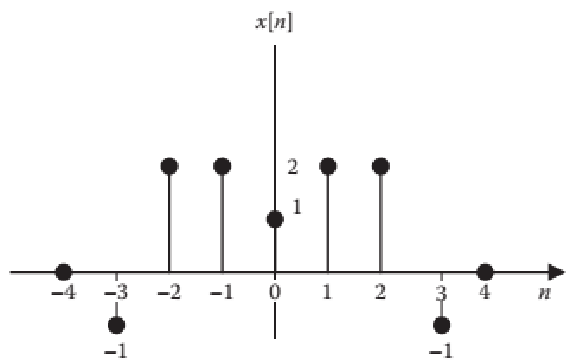


FIGURE 1.62 For Problem 1.32.